Switching Model of a Dynamic Social Network

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***Abstract*—Social media makes possible everyone to speak their mind almost without limitation and restriction. Negative content information such as false statements, fake news, hoax, negative rumors, hate speeches can be stated and can spread easily without obstacles. Harmful negative content information may lead society toward crisis in physical world. Because of that, for a government or a strategic organization, rectifying, suppressing or countering such information is necessary. This paper proposes a model for opinion formation that is subjected to harmful negative content information. The model is derived from previous works in opinion formation. The model applies a switching mechanism to which model that fits a certain condition in society. This model predicts opinions in society after negative content information and social inputs. A simulation shows the society of this switching model mechanism in the presents of such information and social inputs.**

***Index Terms*——Opinion formation, social input, society model**

I. INTRODUCTION

Communication becomes a major aspect of human life. It leads us to achieve, to invent, and to discover successfully what we have known about technological advances, space explorations, good nutrients, human rights, better civilizations, etc. Communication acts as a carrier for ideas, opinions, facts, humors, and principles that lure us into discussion to form new ideas and opinions which definitely affect our responses, judgments, and attitudes toward some particular things, events, phenomenon, or people. As a social agent, we try to communicate with others in daily basis whether it is just to make an interaction or for urgent need. Today, social media applications and websites, such as Facebook, Twitter, Instagram, YouTube, Whatsapp, etc., lead to unrestricted communication. Everyone can speak freely their concerns and opinions about everything. The contents of communication like affections, motivations, advices, knowledge, and useful information are good and are constructive to our society. However, communication does not always carry good things, sometimes negative content information such as wrong or false information, false statements, fake news, hoax, hate speeches, unclarified issues, and rumors are made by irresponsible agent in order to achieve their personal interests. Furthermore, as the freedom of speech and anonymity increase due to the use of social media, society is vulnerable to such information. The harm of negative content information may cause fatality in our society. It can lead to mass panic and social crisis [1] which can end up with decreasing people trusts, war arguments in society, and violent tendency, which in turn may cause the instability to national security and affect the aspects of life of the people. Therefore, communication should be controlled in order to suppress the rumors, to maintain the society, and to recover the public opinions [2], especially for the communication through social media.

Study and analysis of social group hit a breakthrough in 1934 by introducing quantitative approach in describing social relations, called sociometry [3]-[6]. Sociometry advances and becomes a foundation of the social network analysis (SNA) [7]-[9]. SNA extensively uses mathematical methods and algorithmic tools to study the structural properties of social interaction [3], [10]. SNA is used to study broad-range of social aspect such as economics, political studies, and inspires the development of the network theory [11]-[16]. Not only to study social network, the term "cybernetics" which combines system, control, and information theory evolving with social science in 20th century, becoming "sociocybernetics" [3], [18]. Although there has been a lot of studies and analysis in social system, controlling social system with modern control approaches has not been intensively discussed [19]-[21]. Differ with social modern control, there has been work in modelling social network such as French-DeGroot model, Friedkin-Johnsen (FJ) model, Taylor model, and Hegselmann-Krause (HK) model [3], [4], [22], [23], which describe the dynamic of opinion formation in a multiagent system. References [1] and [25] propose about the steps in social crisis in order to recover public opinion and maintain a conducive condition. Reference [1] presents the model and simulation in controlling social under chemical leakage crisis. The formulations for media to answer the questions "when a news should be released?" and "what type of news should be released?" are also proposed.

Although [1] has presented control scheme in social network, individual opinions have not been simulated. References [22], [23], [24], [26], [27] have proposed a model for opinion formation but the social situation for these models

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is considered the same throughout time. This paper shows a dynamic individual opinion when the network is subjected to negative content information by switching the model according to social situation. The switch follows the suitable condition for the model. This paper also shows how the social input such as apologize, rectification, clarification, investigation, and proof presentation shift individual opinion.

In section II, the preliminaries/basic theory and specific problem statement are discussed throughout the paper. Opinion formation model is shown in Section III. Section IV provides a simulation result and analysis. Section V draws conclusions.

II. PRELIMINARIES

A. Opinion, Agent, and Matrix Form

Opinion is a cognitive orientation toward some objects (issue, event, or individual). For examples, displayed attitudes or subjective certainties or beliefs [26]-[30]. As a result of a social interaction, individual opinion evolves throughout the time. Although opinion is qualitative entities, many researcher try to quantitatively model opinion by scalar or vector value. Until now, theoretic studies have been proposed in various dynamic opinion model such as in [22]-[27]. These models deal with real-valued opinion which is scaled in certain range, e.g., [0, 1] or [–1 , 1]. The models also focus on the dynamic of individual opinion as a social actor or "agent". It should be mentioned that all of the model above are idealistic closed community in which the number of agent in community is fixed, cannot decrease or increase.

Dynamic of individual opinion is affected by other opinions and social bonding in the society. e.g., a person's opinion is a result of opinion interaction between him/her, the parents, friends, teachers, and others. However, not all opinion interaction is weighted the same. Some agents might be weighted more than the others. This weighted process happens because of social bonding that has been formed on an agent. Social bonding is described mathematically with a directed graph or simply a graph.

Definition 1 (Graph). A graph is a pair $G = (V, E)$, where $V = \{v_1, \ldots, v_n\}$, n is the number of agents, and $E \subseteq V \times V$ are finite sets. The elements v_i are called vertices or nodes of G and the elements of E are referred to as its edges or arcs.

The connection strength between two agents can be defined by weighting the edge by some real positive value. For convenient use, the graph can also be represented in a matrix form.

Definition 2 (Matrix form of graph)**.** Given a graph $G = (V, E)$, a nonnegative matrix $W = (w_{ij})_{i,j \in V}$ is adapted to G or is a matrix for G if $(i, j) \in E$ when $w_{ij} > 0$ and $(i, j) \notin E$ otherwise.

The social bonding between agent-*i* and –*j* is represented by w_{ij} which acts as a weighting factor of agent *i* to agent *j*. Or, it can be seen as a percentage of agent-*j*'s opinion that assimilates with the agent-*i*. Intuitively, for each agent, the sum of weighting factor to other agent is 1.

Definition 3 (Stochasticity and sub stochasticity). A nonnegative matrix W (not necessarily square) is called *stochastic* if all its rows sum to 1 (i.e., $\sum w_{ij} = 1, \quad \forall i$ and *substochastic* if the sum of each row is no greater than 1 (i.e., $\sum w_{ij} \le 1, \quad \forall i$).

B. French-DeGroot and Friedkin-Johnsen (FJ) Model

French-DeGroot model is one of the first agent-based models of opinion formation which is considered as an iterative formation [22]. Suppose there are n agents in society with their initial opinion $x_1, x_2, \ldots, x_n$, respectively. At some iteration k, each individual opinion form an opinion vector $x(k) = \{x_1(k)\ x_2(k)\ \ldots\ x_n(k)\}^T$, $k = 0, 1, \ldots, m$. Assume that their social bonding has been obtained and presented in the form of matrix W which is n-by-n stochastic matrix. French-DeGroot model follows a simple iterative model

$$x(k+1) = W\,x(k),\ k = 0, 1, \ldots, m, \tag{1}$$

which is equivalent to

$$x_i(k+1) = \sum w_{ij} x_j(k), \quad \forall i, \quad k = 0,1,\ldots,m\,. \tag{2}$$

Note that w_{ij} can be seen as how much agent *i* believes on agent *j* (or, the influence of agent *j* toward the agent *i*). High value of w_{ij} means that the agent *i* has high believe in the agent *j* (or, the agent *j* has high influence toward the agent *i*). w_{ii} is self-believed of agent *i*. High value of w_{ii} described that agent *i* is highly weighted his self-opinion and almost unaffected by other opinions. An extreme case 1 where $w_{ii} = 1$ means that the agent *i* is "stubborn agent", in which, its opinion cannot be changed throughout social interaction. In case 2, when $w_{ij} = 1$, it means that the agent *i* "worship" the agent *j*.

Friedkin-Johnsen in [23] proposes a slight modification in French-DeGroot model by adding prejudice opinion is proposed. The modification introduces a diagonal matrix $\Lambda = \mathrm{diag}(\lambda_1\ \lambda_2\ \ldots\ \lambda_n)$, where $\lambda_i \in [0,1]$, and prejudiced vector u. λ_i describes the susceptibility of the agent *i* to social process in society and the prejudiced vector u is a constant vector describing agent anchored opinion (their believes or principles). Friedkin-Johnsen (FJ) opinion model follows

$$x_i(k+1) = \Lambda W x(k) + (1-\Lambda)u\,. \tag{3}$$

C. Hegselmann-Krause (HK) Model

Hegselmann-Krause in [27] models the opinion formation, not based on the social bonding of each agent as proposed by French-DeGroot and Friedkin-Johnsen, but the social bonding only occurs when two or more agent's opinion only differ in a small proportion. The Hegselmann-Krause (HK) model leads that, toward discussions, group of agents result in consensus in a subgroup of society. This model is based from a phenomenon of "same bird flock together" and this also happens in our society. For example, a person who likes pop music tends to interact with others who also like pop music, creating a forum discussion that results in consensus (how good pop music is). In other words, similar people are more likely to have interaction rather than dissimilar people. In the model formulation, [27] defines fix bounded confident $d > 0$ called set of $\{x_i - d < x < x_i + d\} \in R$ confidence interval of agent *i*. According to HK model, the agent only interacts with other agents, in which, their opinions are located in this confidence interval and himself. Define $I_i(x) = \{j: |x_j - x_i| \le d\} \in \{i\}$ and $|I_i(x)|$ is the trusted individual of the agent *i* and their members. Opinion in HK evolves as follows

$$x_i(k+1) = \frac{1}{|I_i(x(k))|} \sum_{j \in I_i(x(k))} x_j(k), \quad \forall i \,. \tag{4}$$

Thus, the weighted matrix is defined as

$$w_{ij}(x) = \begin{cases} \dfrac{1}{|I_i(x(k))|}, & \text{if } j \in I_i(x(k)) \\ 0, & \text{otherwise.} \end{cases} \tag{5}$$

Note that the matrix *W* in HK model is not constant as in French-DeGroot or FJ model. The matrix *W* changes overtime to make sure that social bonding only happens when opinion difference is in confidence interval. This model describes very well the discussion aspect in society. Bounded confidence represents how open the society to discussion. High value of *d* means that the society are welcome to discussion even when the agents has large amount of different in opinion.

D. Problem Statement

Social media makes all kind of communication possible, from a positive discussion to a negative one. An agent can spread negative content information to attack someone (a person, an organization, or a government) in order to shift society opinions. For the purpose of our discussions, an agent who spreads a rumor, which can have positive content or negative one, is called the "rumor agent" and an agent which is attacked by the rumor is named the "attacked agent". Notice that spreading of this kind of rumor (statement) should be rectified, suppressed or countered in order to maintain the society. In a contrast, if the rumor is not suppressed, false statement can lead to social crisis and harms society principles. This work tries to combine FJ and HK model and proposed switching model to describe opinion formation when negative content information is happening in order to see the dynamic of individual opinion.

III. Switching Model of a Social Network

The FJ model in (3) fits very nicely with our hypothesis of opinion formation. The model states that an individual opinion is based on weighting every opinion in the society and their own prejudiced. But, this model does not employ entities, such as apologize, rectifcation, clarification, help, campaign, etc., made by some agents.

Definition 4 (social input) Social input is a publication of action which has an impact in society such as apologize, rectifcation, clarification, announcement, investigation, help, campaign, or others, in order to control public opinion.

Social inputs happen in reality, e.g., apologize certain agent in mass media, current report of some investigation case, presenting data to clarify misconception, and so on. With the same basic idea of weighting every information that is use in original FJ model, additional of social input *s* modify FJ model as follows

$$x_i(k+1) = \Lambda_1 W x(k) + \Lambda_2 s + \Lambda_3 u, \quad \Lambda_3 = 1 - \Lambda_1 - \Lambda_2 \,. \tag{6}$$

Λ_l, l = 1, 2, 3, are non-negative diagonal matrices, in which, their sum make and identity matrix, $\Lambda_1 = \text{diag}(a_1 \; \ldots \; a_n)$, $\Lambda_2 = \text{diag}(b_1 \; \ldots \; b_n)$, where a_i and b_i represent susceptibility of the agent *i* toward social interaction and social input.

Even when the social input has been included in the modification of the FJ model, this model has not described opinion assimilation. In reality, there are tons of forum and discussion (face to face or social media) which result in consensus within some agents. HK model in (4) and (5) describes this assimilation. Taking an account of every new aspect in social activity such as weighting, social input, and opinion assimilation, this paper proposes a hybrid model to describe a society when subjected to an attacking rumor. The proposed model is the switching from one model to another according to the situation. We introduce the society phases:

Initial phase: The initial phase is best described by FJ model in (3) because there is no attack and no social input yet. In this phase, the society forms the opinion based on the social interaction and their own prejudice.

Attack phase: When rumor agent states a negative rumor, society starts to weight the attacked agent's reaction, what an agent should say, do, or publish. In this stage, social input has been introduced. Therefore, the model switches from the FJ model in (3) to the modified FJ model in (6).

Discussion phase: This phase starts when the attacked agent has finished publishing its social input. The society now can start a valid discussion based on what have been presented by the attacked agent. Notice that the discussion does not only happen in the discussion phase, it can also occurs during the

initial and attack phase. However, the discussion in this phase will not lead to a consensus because the discussion doesn't have a valid material from the attacked agent. In this discussion phase, the model switch to HK model with certain boundary of confidence *d* as in (3) and (4). The value of *d* represents the strength of facts that have been presented by the attacked agent. High value of *d* means that the attacked agent provide sufficient data in wide range of discussion.

Final phase: This phase is the final state when each agent opinion converges to their equilibriums.

IV. Simulation Result and Analysis

In order to understand the proposed switching model, a numerical simulation is presented. Suppose there are seven agents in the society, and the number of agents is fixed. All agents have their initial opinions about something (a person, organization, government, or others) and every agent is free to communicate with others. Because of the communication and discussion within this group, their opinions have some dynamics. The opinion of all agents can change over time.

The simulation of opinion formation is depicted in Fig. 1. The opinion is scaled in the range of [0, 1], where 0 means very bad opinion, 1 means very good opinion, and 0,5 means neutral. The weighted of agents are defined in three groups that are: 1) the affected agents are the ones that are more belief in a rumor; 2) the intermediate agents are the neutral ones that the social input and the rumor affect equally; 3) the restored agents are the ones that have more belief in social input.

In the initial phase, the information sources are only from the other agents' opinion and their own prejudices, then the FJ model in (3) is chosen to describe the communication in this phase. Assume that at time t_a, there is a rumor agent in this society that spreads a negative rumor indicating by significant decrease in its opinion closer to the value of 0. The phase then enters the attack phase.

In the attack phase, each agent in society is firstly affected by this rumor and their opinions decrease. As a response, the attacked agent reacts to this rumor by making a statement or social input (approve, apologize, ignore, or deny). Furthermore, the attacked agent can also release another social input if necessary. In the simulation, we assume that the social input from the attacked agents can increase the opinion which affects other agents in the society. Then, after a while the society has another source of information which comes from social input of attacked agents. Such information may include clarification, rectification, investigation, and proof presentation of facts that affects the society stronger so that the opinion of the society can increase further. Additional social input makes model switch to modified FJ model in (6).

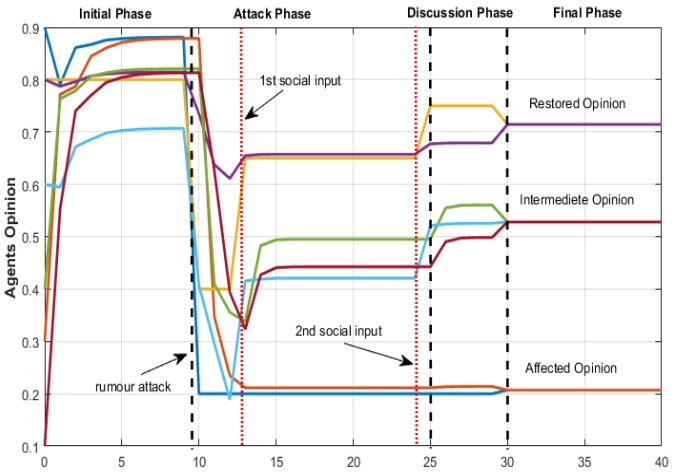

Fig 1. Opinion formation toward attacked agent when subjected to rumor and social inputs (should add information about each phase in this graph to make it more understandable).

After some times, when attacked agent has finished to present clarification, apologize, facts, etc (social input) to society, the model enters discussion phase. Consider that every agent in society have enough discussion material (coming from rumor agent and attacked agent), opinion assimilation is happening in discussion phase. Dynamic of opinion in this phase is best described by HK model, therefore in this phase, the opinion formation model dynamics is switched to the HK model. For some nonnegative, square, and stochastic weighting matrix *W*, susceptibility matrices Λ_1, Λ_2 and a bounded confidence *d*.

From the simulation scenario, the equilibriums of the individual phases are discussed as follows. In the initial phase, each agent interacts with each other resulting an equilibrium opinion or consensus.

Definition5 (initial equilibrium opinion) $x^{\text{init,eq}} = \{x_1^{\text{init,eq}} x_2^{\text{init,eq}} \dots x_n^{\text{init,eq}}\}$ is vector of equilibrium opinion in the initial phase where the rumor agent has not attacked yet and there is no social input.

After the rumor agent attacks, overall opinions in society decrease. But, not every agent opinion decreases significantly because there are still some agents that do not heavily weight to the rumor agent's opinion. Entering the attack phase, the attacked agent releases two social inputs in order to restore public opinion. Although the social inputs that had been published do not fully restore each agent opinion, it can still affect some agents to recover their opinions toward the rumor given by the attacked agent. Finally, when sufficient data has been published, the society enters the discussion phase which assimilates neighboring opinions to subgroup a consensus.

TABLE 1.
INITIAL OPINION, INITIAL EQUILIBRIUM OPINION, FINAL EQUILIBRIUM OPINION, AND OVERALL OPINION CHANGE OF THE SIMULATION

Agent No.	Initial Opinion	Initial Equilibrium	Final Equilibrium	Overall Change
1	0.9	0.88	0.20 (rumor agent)	-0.68
2	0.3	0.87	0.20 (affected agent)	-0.67
3	0.8	0.80	0.71 (restored agent)	-0.08
4	0.8	0.81	0.71 (restored agent)	-0.10
5	0.4	0.82	0.52 (intermediate agent)	-0.30
6	0.6	0.70	0.52 (intermediate agent)	-0.18
7	0.1	0.81	0.52 (intermediate agent)	-0.29

Definition 6 (final equilibrium opinion) $x^{\text{fin,eq}} = \{x_1^{\text{fin,eq}} \; x_2^{\text{fin,eq}} \; \dots \; x_n^{\text{fin,eq}}\}$ is vector of equilibrium opinion in the final phase where opinion assimilation has been attained.

From the final phase, the final opinions are converged into three categories which are affected agents, intermediate agents, and restored agents. Affected agents are agents in which their opinions are not recovered and have significant decreasing opinion in the initial phase. Intermediate agents are agents which the opinion are affected by rumor and social inputs quite even. The restored agents are agents which their opinions are more affected by social inputs rather than the rumor. In order to analyze the overall opinion changes after the society has subjected to rumor and social input, simply substract $x^{\text{fin,eq}}$ with $x^{\text{init,eq}}$ to obtain overall opinion change.

Definition 7 (overall opinion change) The overall opinion change is the difference of final equilibrium phase with initial equilibrium phase, i.e., $\Delta x = x^{\text{fin,eq}} - x^{\text{init,eq}}$.

Overall opinion change represents how rumors and social inputs affect each individual agent in society. The negative value of Δx_i describes that agent i is more affected by rumor compared to the social input. The positive one describes that agent i is more affected by social input rather than rumor.

From the simulation result in Fig. 1, it is shown that each model is suitable in different phase. In the initial phase, when the rumor has not been issued, the agent's opinion is determined by the FJ model and the opinion dynamics are based on the weight of beliefs to other agents and itself. When a rumor attacks by a rumor agent, there exists a negative social input. By relating to model (6), the agents' opinions decrease depending on the value of agents' beliefs, susceptibility matrices $\Lambda_1(t_{att,1})$ and $\Lambda_2(t_{att,1})$, where $t_{att,1}$ is the time when the rumor attack happens, the agent anchored opinion u, and also the strength of rumor attack input $s(t_{att,1})$. If the values of Λ_1 and Λ_2 are low and attack input $s(t_{att,1})$ is high, the opinion of agent is highly affected, and vice versa. This also happens for the positive social inputs where the model used is (6); the different with negative social input is that the input s drives the opinion of the agents to increase. In the decision phase, the model applies HK model, where the opinion of a group that has the same confidence arrives in consensus.

From the attack phase of opinion dynamics, we can conclude that the opinion dynamics in regards to social inputs are depended on the agents' beliefs, the agent anchored opinion, the strength of social inputs, and the susceptibility matrices. We can also conclude that the switching model, with the HK model is used in the discussion phase, render the opinion agents into three opinion categories which are affected, intermediate, and restored opinion. In this simulation, the agents with restored opinion are the agents that are the least affected, while the restored agents are the most affected by the negative rumor.

The initial opinion, initial equilibrium opinion, and final opinion for each agent in this simulation is tabulated in Table 1. When the negative rumor has not been issued, the agents' initial opinion can be attracted to higher values since there is no discussion on the issue. The agent 2 with high initial opinion equilibrium can be affected with the final equilibrium of 0.20. The overall change of the affected agent's opinion is –0,67, which is close to the rumor agent. This is due to the belief of agent 2 to agent 1 (the rumor agent) is high and so are the susceptibility matrices of Λ_1 and Λ_2. Thus, the values of Λ_3 and the social input s cannot raise the agent's opinion to when the initial equilibrium is achieved. The restored agents 3 and 4 have high beliefs to each other, and have low belief to the rumor agent. The susceptibility matrix Λ_1 and Λ_2 are high to the positive anchored opinion. Therefore, when the rumor agent attacks, the opinion is slightly affected. Thus, the overall change of the restored agents are very small. The intermediate agents 5, 6, and 7 have moderate values of W and the susceptibility matrices of Λ_1 and Λ_2. From Fig. 1, it can be seen that the intermediate agents can be highly affected by the rumor agents, but they also can return to the moderate values around 0.52 after positive social inputs and discussion phase. It is worth to notice that the result of this model is heavily depended on matrix W and susceptibility matrices Λ_1 and Λ_2 which represent the characteristics of the agents in a society.

Identification of these parameters (W, Λ_1 Λ_2) are needed to be further investigated. In reality, initial equation, final equation, and overall change data can be gathered by surveying each agent's opinion before and after certain issue, rumor, or negative information.

V. CONCLUSION

This paper proposes a switching model to extent opinion formation in society when some negative rumors are happening. Introducing society phases which are initial phase, attack phase, discussion phase, and final phase. The model is switched as the society phase changes. The initial phase is when rumor agent has not attacked yet and describe by FJ model. Adding some social inputs in attack phase make the

model switch to modified FJ model where the social inputs, prejudices, and the communication among agents are interacted. Finally at discussion phase, the society assimilates their opinion to reach some consensus with their neighboring opinions. Defining the initial and final equilibrium phase, overall opinion shifts can be analyzed by calculating the difference of equilibriums. Each component of the equilibrium difference represents how each individual affected by rumor and social input in general. It can be observed from the simulation that rumor has negative effect to society opinion (opinions decrease for all agents). To suppress or counter this rumor, social inputs such as apologize, rectification, clarification, or investigation is published to the society. Social input brings positive contribution to recover public opinion, notice that, not all agents are heavily affected by rumor or social input. In the simulation, three final equilibrium opinions are obtained which are affected opinion (affected agents), intermediate opinion (intermediate agents), and restored opinion (restored agents).

Further research will include the conformity of the switch model with the reality. Also, the identification of the parameters of the weighted matrix needs to be investigated.

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